Probability:

Complementary Events

Addition rule

Complementary Events

the events of one outcome happening and that outcomes not happening are complementary (opposite)
(not E is contrary event to E)

For example : you pick up a card from a deck

E:→ P(Heart)= $\frac{1}{4}$

Not E: → P(not Heart)= $\frac{3}{4}$

Complementary Events

THE SUM of the of the probabilities of complementary events is 1.

$$P(E)+P(\overline{E})=1$$

Not E

from which I get:

The Probability of the contrary event Not E is:
"1 minus the Probability of the event E"

$$P(\overline{E})=1-P(E)$$

Not E

example

Complementary Events

You pick up a card from deck of 52 cards.
Which is the probability of **picking** a figures?

$$P(figures) = \frac{12}{52}$$

Which is the probability of **not picking** a figures ?

$$P(NotFigures) = 1 - \frac{12}{52} = \frac{52-12}{52} = \frac{40}{52}$$

Examples

Complementary Events

1) The probability that it will rain tomorrow is 0.4 .
What is the probability it does not rain ?

P(not Rain) = 1-0.4 = 0.6

2) Tossing 2 coins,which is the probability of:

a) never getting Tail ?

P(never T) = P(Head Head) = 1/4

b) getting at least once Tail?

(TT or HT or TH)

P(at Least Once T)=1-P(never T)=1-1/4= 3/4

ADDITION RULE

PROBABILITY OF A OR B

There are two situations

1) Disjoint Events

2) NOT Disjoint Events

what does it mean

Disjoint Events ?

Two events are Disjoint (Mutually Exclusive) if they

can't happen at the same time

- **Turning left and turning right** are Mutually Exclusive (you can't do both at the same time)
- Cards: **Kings and Aces** are disjoint

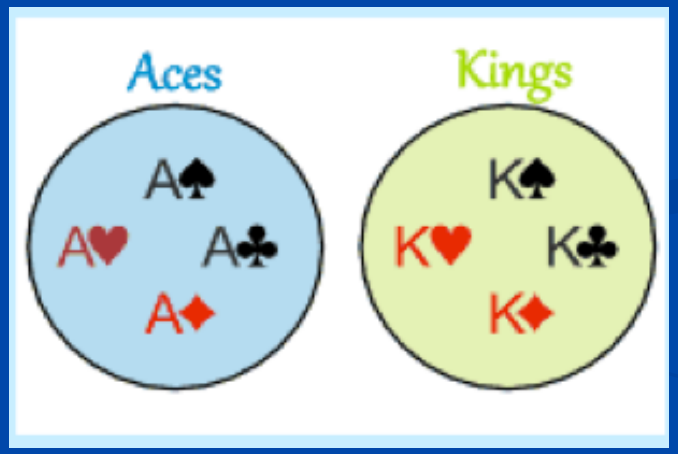
What is Not Disjoint (not Mutually Exclusive) ?

- **Turning left and scratching your head** can happen at the same time
- Cards: **Kings and Hearts**, because we can have a King of Hearts!

1) DISJOINT Events (Mutually Exclusive)

example

A single card is chosen at random from a standard deck of 52 playing cards.
What is the probability of choosing an Ace or a King?

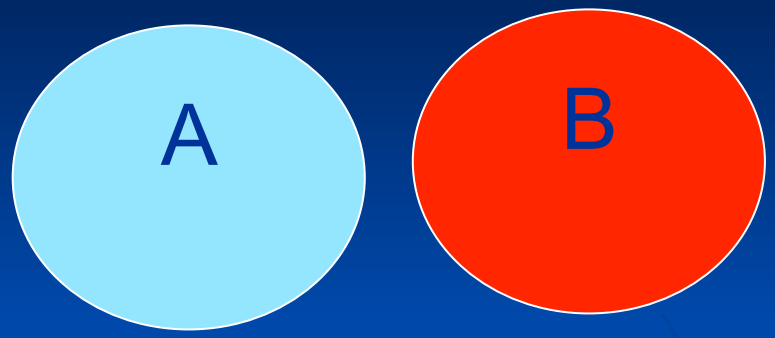


P(ACE or KING) = P(Ace) + P(King)

= 4/52 + 4/52 = 8/52

1) DISJOINT events

Addition Rule for DISJOINT Events:



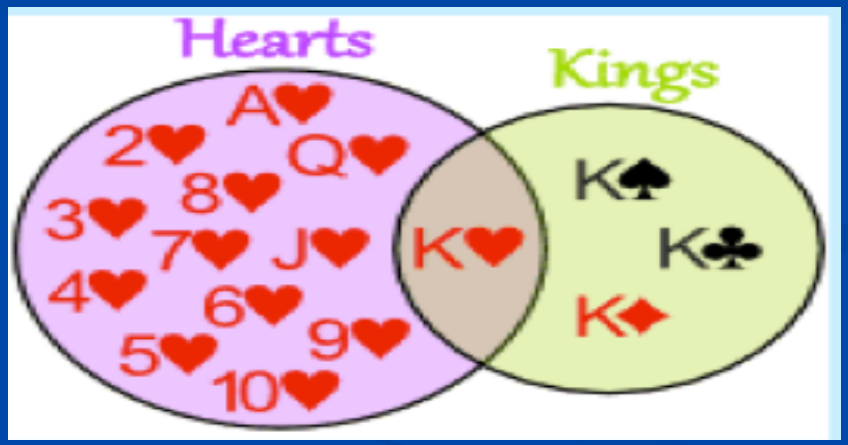
When two events A and B are disjoint, the probability that A or B will occur is:
the SUM of the Probability of each Event.

$$P(A \text{ or } B) = P(A) + P(B)$$

2) NOT DISJOINT (NOT Mutually Exclusive)

example

A single card is chosen at random from a standard deck of 52 playing cards.
What is the probability of choosing an Heart or a King?

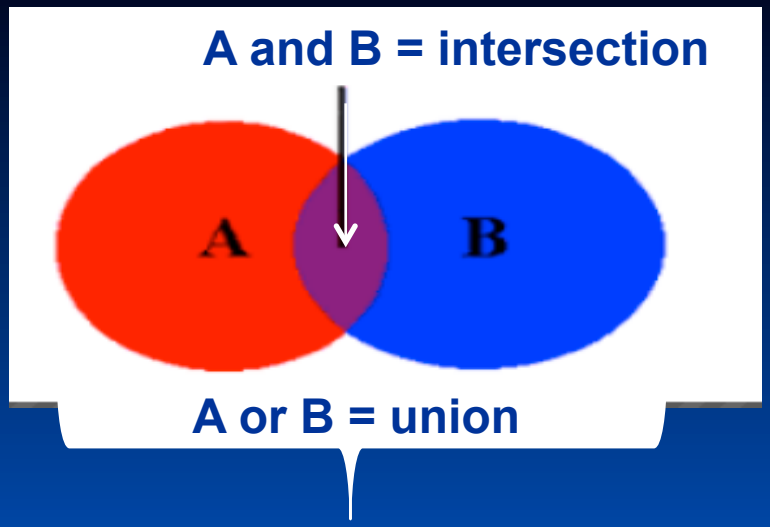


P(H **or** K) = P(H) + P(K) - P(both)

=13/52 + 4/52 –1/52 = 16/52

1) NOT DISJOINT

ADDITION RULE for NOT disjoint Events



When two events A and B are NOT DISJOINT, the probability that A or B will occur is : the SUM of the probability of each event, MINUS the probability of the overlap.

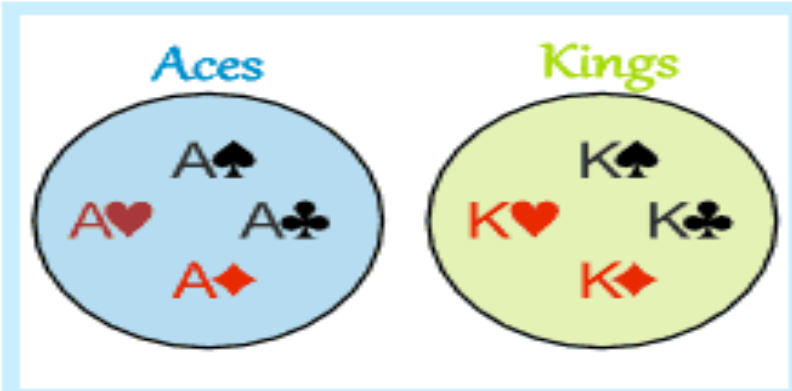
both

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

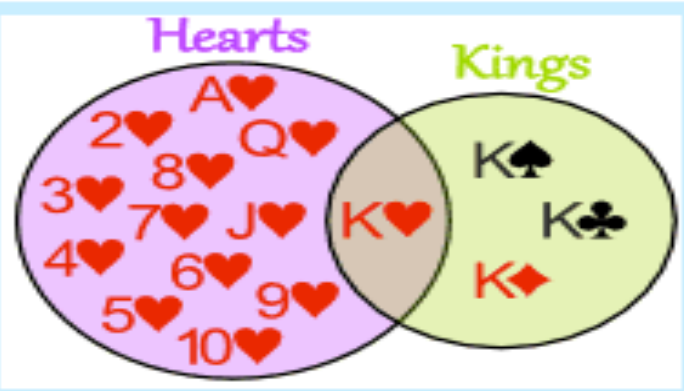
U union

∩ Intersection

SUMMARY : ADDITION RULE



Aces and Kings are
Mutually Exclusive
(can't be both)



Hearts and Kings are
not Mutually Exclusive
(can be both)

DISJOINT EVENTS

Mutually Exclusive

A **and** B together is impossible:
P(A and B) = 0

P(A or B) = P(A) + P(B)

NOT DISJOINT EVENTS

Not Mutually Exclusive

A and B together is possible !

P(A or B) = P(A) + P(B) – P(A and B)

TRY IT YOURSELF

TEST

1: A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing an Ace or a figure?

2: A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing an Ace or Red Card?

3: You are going to roll two dice. Find: P(sum that is even or sum that is a multiple of 3).

ANSWER 1

1: A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing an Ace or a figure?

P(Ace)=4/52

P(Figure)=12/52

- **These events are mutually exclusive (disjoint) since they cannot occur at the same time.**

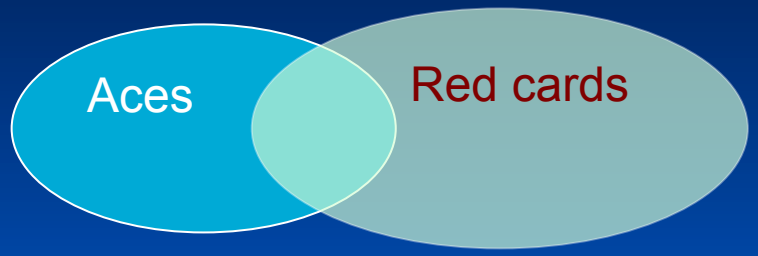
$$P(A \text{ or } B) = P(A) + P(B)$$

U union

P(Ace OR Figure) = 4/52+12/52 = **16/52**

ANSWER 2

2. A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing an Ace or Red Card?



P(Ace)=4/52

P(Red card)=26/52

- **These events are NOT disjoint since they have some overlap (favorable outcomes in common)**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

U union

∩ Intersection

P(Ace OR Red Card) = 4/52+26/52-2/52 = 28/52

ANSWER 3

3. You are going to roll two dice. Find P(sum that is even or sum that is a multiple of 3).

The addition rule says we need to find

P(even) + P(multiple of 3) - P(both)

The number of possible outcomes of rolling two dice = 36

P(even) means how many ways to roll:2, 4, 6, 8, 10, or 12.

P(even) = 18/36

P(multiple of 3) means how many ways to roll : 3, 6, 9 or 12.

P(multiple of 3) = 12/36

P(both) means what is the overlap. Notice that 6 and 12 occur in both places and have been counted twice. We need to subtract those out.→ **P(both) = 6/36**

P(even or multiple of 3)= 18/36 + 12/36 - 6/36 = 24/36