CORREZIONE COMPITI AGOSTO 2021 - 4SSA

1 -DOMINIO

$y = \frac{x^2 - 4}{3x-6}$ funzione razionale fratta $\quad 3x-6 \neq 0 \quad 3x \neq 6$ $\quad x \neq 2$

$y = \frac{x-2}{x^2-3x+2}$ funzione razionale fratta $\quad x^2-3x+2 \neq 0 \quad x \neq \frac{3 \pm \sqrt{9-8}}{2}$ $\begin{cases} \frac{3-1}{2} = 1 \\ \frac{3+1}{2} = 2 \end{cases}$ $x \neq 1$ e $x \neq 2$

$y = \sqrt{x-5}$ funzione irrazionale intera $\quad x - 5 \geq 0$ $\quad x \geq 5$

$y = \sqrt{4x^2 - x}$ f. irrazionale intera $\quad 4x^2 - x \geq 0 \quad x(4x-1) = 0$ $\begin{cases} 0 \\ \frac{1}{4} \end{cases}$ $x \leq 0 \vee x \geq \frac{1}{4}$

$y = \sqrt{x^2-1}$ f. irrazionale intera $\quad x^2 - 1 \geq 0 \quad x = \pm 1$ $\quad x \leq -1 \vee x \geq 1$

$y = \ln(x-4)$ f. logaritmica $\quad x - 4 > 0$ $\quad x > 4$

$y = \ln(x^2+6x)$ f. logaritmica $\quad x^2+6x > 0 \quad x(x+6) = 0$ $\begin{cases} x = 0 \\ x = -6 \end{cases}$ $x < -6 \vee x > 0$

2 - DERIVATA PRIMA

$y = 4x^3 - 5x^2 + 7x + 3$ $\quad y' = 12x^2 - 10x + 7$

$y = \ln x + 4$ $\quad y' = \frac{1}{x}$

$y = e^x + 4x + 6$ $\quad y' = e^x + 4$

$y = \operatorname{sen} x + 3\cos x$ $\quad y' = \cos x - 3\operatorname{sen} x$

$y = (5x-2)(x+1)$ $\quad y' = D[5x-2](x+1) + (5x-2)\cdot D[x+1]$

$= [5](x+1) + (5x-2)\cdot[1]$

$= 5x + 5 + 5x - 2 = 10x + 3$

$y = (x^2+9x)\cdot(x-3)$ $\quad y' = D[x^2+9x]\cdot(x-3) + (x^2+9x)\cdot D[x-3]$

$= [2x+9]\cdot(x+3) + (x^2+9x)\cdot[1]$

$= 2x^2 + 6x + 9x + 27 + x^2 + 9x$ $\quad = 3x^2 + 24x + 27$

$$y = \frac{x^2-2x}{x^2-7x} \qquad y' = \frac{D[x^2-2x]\cdot(x^2-7x) - (x^2-2x)\cdot D[x^2-7x]}{(x^2-7x)^2}$$

$$= \frac{[2x-2](x^2-7x) - (x^2-2x)\cdot[2x-7]}{(x^2-7x)^2}$$

$$= \frac{2x^3-14x-2x^2+14x-2x^3+7x^2+4x^2-14x}{(x^2-7x)^2} = \boxed{\frac{9x^2-14x}{(x^2-7x)^2}}$$

$$y = \frac{3x^2-x}{x^2+8x} \qquad y' = \frac{D[3x^2-x]\cdot(x^2+8x) - (3x^2-x)\cdot D[x^2+8x]}{(x^2+8x)^2}$$

$$y' = \frac{[6x-1](x^2+8x) - (3x^2-x)\cdot[2x+8]}{(x^2+8x)^2}$$

$$y' = \frac{6x^3+48x^2-x^2-8x-6x^3-24x^2+2x^2+8x}{(x^2+8x)^2} = \boxed{\frac{21x^2}{(x^2+8x)^2}}$$

3- RICERCA DEI PUNTI STAZIONARI (verifica con GeoGebra)

$y = x^2 + x - 2$ **parabola con concavità verso l'alto**

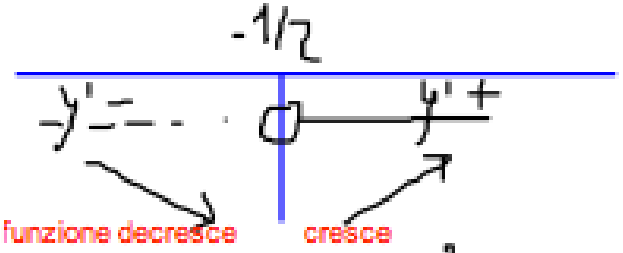
DOMINIO $\forall x \in R$ o $(-\infty ; +\infty)$

DERIVATA PRIMA $y' = 2x + 1$

PUNTI STAZIONARI Y'=0 $2x + 1 = 0 \quad 2x = -1 \quad x = -1/2$

MONOTONIA Y'>0 $2x + 1 > 0 \quad x > -\frac{1}{2}$

GRAFICO



$MIN = \left(-\frac{1}{2}; \dots\right)$

TROVO ORDINATA/E $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 2$

$\frac{1}{4} - \frac{1}{2} - 2 = \frac{1 - 2 - 8}{4} = -\frac{9}{4}$

RISPOSTA $MIN = \left(-\frac{1}{2} ; -\frac{9}{4}\right)$

$y = -x^2 + 4x$ **parabola con concavità verso il basso**

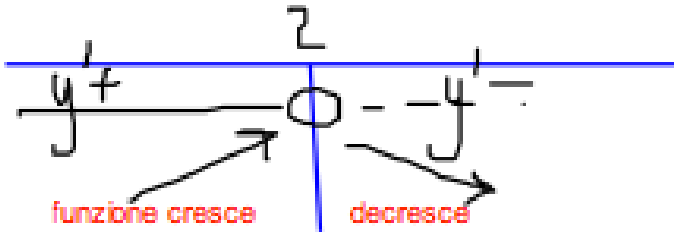
DOMINIO $\forall x \in \mathbb{R}$

DERIVATA PRIMA $y' = -2x + 4$

PUNTI STAZIONARI Y'=0 $-2x+4 = 0 \quad 2x-4=0 \quad 2x=4 \quad x=2$

MONOTONIA Y'>0 $-2x+4>0 \quad 2x-4<0 \quad 2x<4 \quad x<2$

GRAFICO



$MAX = (2; \dots)$

TROVO ORDINATA/E $f(2) = -(2)^2 + 4(2) = -4 + 8 = 4$

RISPOSTA $MAX = (2; 4)$

$y = x^3 + 3x^2$ **CUBICA "crescente "**

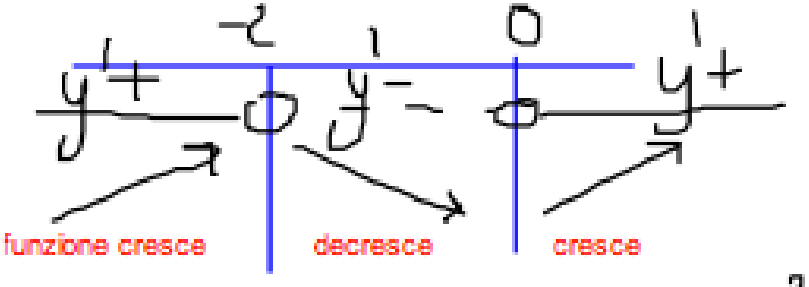
DOMINIO $\forall x \in R$

DERIVATA PRIMA $y' = 3x^2 + 6x$

PUNTI STAZIONARI Y'=0 $3x^2 + 6x = 0 \quad x(3x+6) = 0 \rightarrow x = 0 \;\; x = -2$

MONOTONIA Y'>0 $3x^2 + 6x > 0 \quad x = 0, -2 \quad \Delta > 0$ CONC $x < -2 \vee x > 0$

GRAFICO



MAX = (−2; ...)
MIN = (0; ...)

TROVO ORDINATA/E

$f(-2) = (-2)^3 + 3(-2)^2 = -8 + 12 = +4$

$f(0) = (0)^3 + 3(0)^2 = 0$

RISPOSTA $MAx = (-2; 4) \quad MIN = (0; 0)$

$y = -\frac{1}{3}x^3 + 2x^2 - 3x$ **CUBICA " decrescente "**

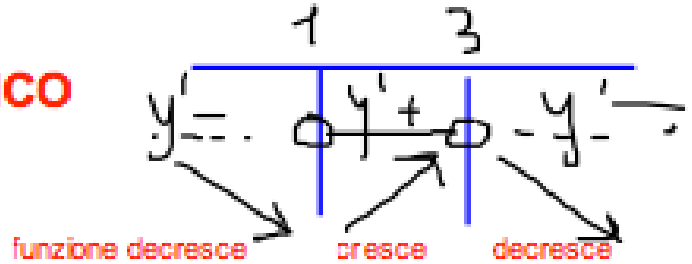
DOMINIO $(-\infty; +\infty)$

DERIVATA PRIMA $y' = -\frac{1}{3} \cdot 3x^2 + 2 \cdot 2x - 3 \cdot 1 = -x^2 + 4x - 3$

PUNTI STAZIONARI Y'=0 $-x^2 + 4x - 3 = 0 \quad x^2 - 4x + 3 = 0 \quad x = \frac{4 \pm \sqrt{16-12}}{2} = \begin{cases} \frac{4-2}{2} = 1 \\ \frac{4+2}{2} = 3 \end{cases}$

MONOTONIA Y'>0 $-x^2 + 4x - 3 > 0 \quad \begin{matrix} 1 \\ = 0 \\ 3 \end{matrix} \quad \Delta > 0$ DISC $\quad 1 < x < 3$

GRAFICO



$MIN = (1; \dots)$

$MAX = (3; \dots)$

TROVO ORDINATA/E

$f(1) = -\frac{1}{3}(1)^3 + 2(1)^2 - 3(1) = -\frac{1}{3} + 2 - 3 = \frac{-1+6-9}{3} = -\frac{4}{3}$

$f(3) = -\frac{1}{3}(3)^3 + 2(3)^2 - 3(3) = -9 + 18 - 9 = 0$

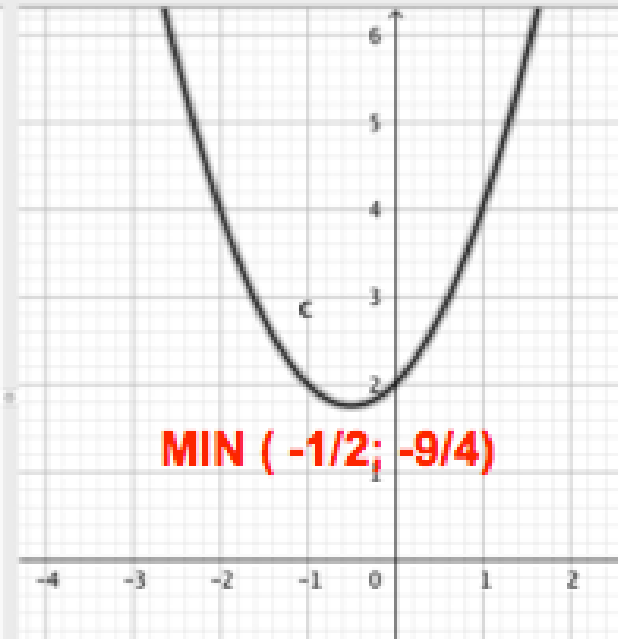
RISPOSTA $MIN = \left(1; -\frac{4}{3}\right) \quad MAX = (3; 0)$

VERIFICA CON GEOGEBRA

Conica

c: $y = x^2 + x + 2$

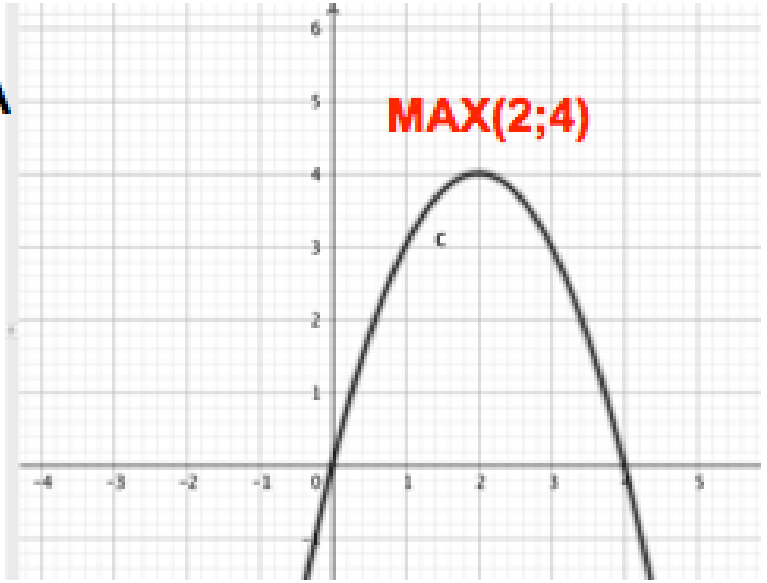
PARABOLA



Conica

c: $y = -x^2 + 4x$

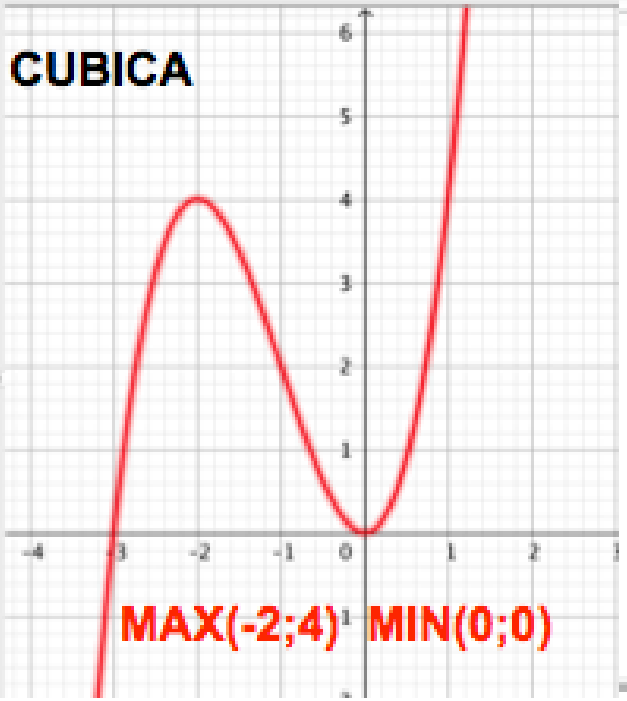
PARABOLA



Funzione

f: $y = x^3 + 3x^2$

CUBICA



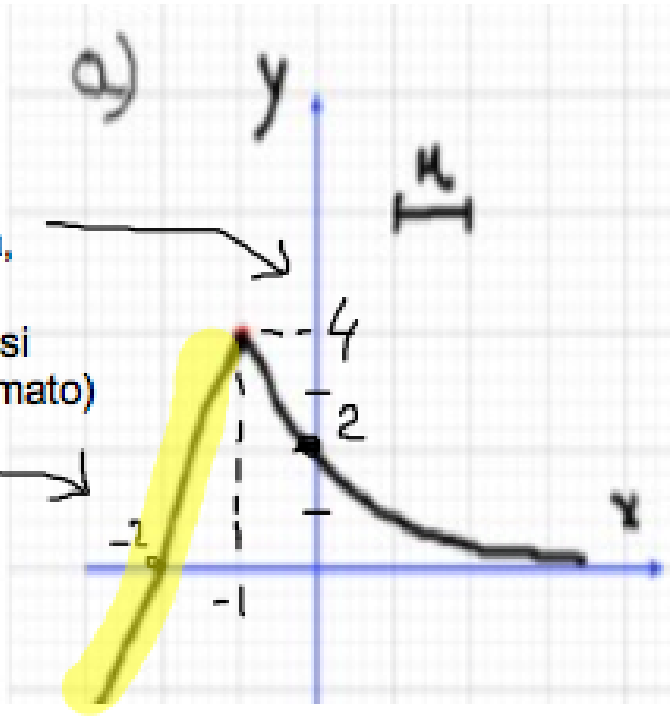
Funzione

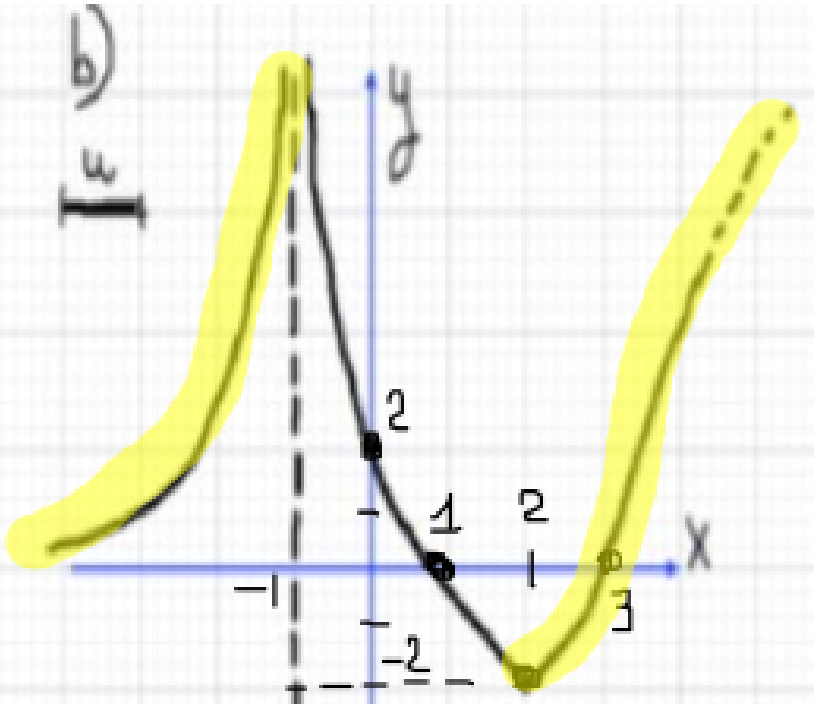
f: $y = -\frac{1}{3}x^3 + 2x^2 - 3x$

CUBICA

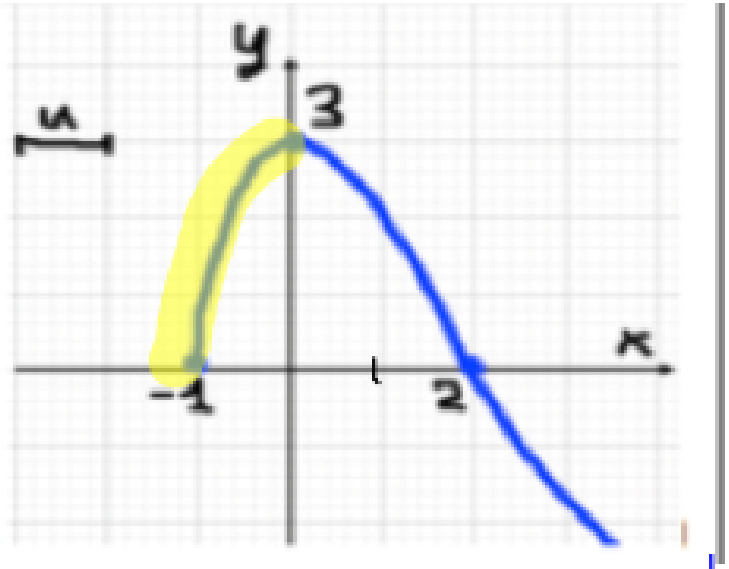
4-ANALISI GRAFICO

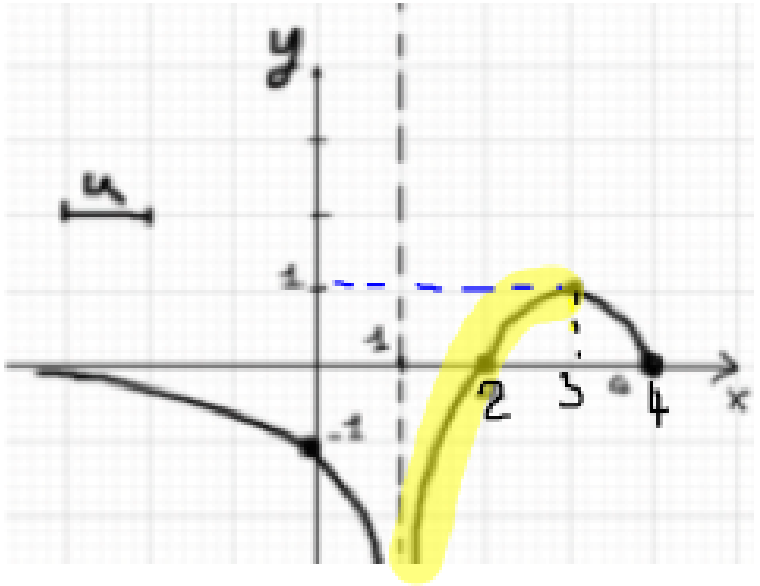
per prima cosa
in base all'unità di misura,
segno nel grafico
i valori mancanti sugli assi
(anche in modo approssimato)





DOMINIO	$\forall x \in R$	$x \neq -1$
CODOMINIO	$y \leq 4$	$y \geq -2$
SEGNO $f(x) > 0$	$x > 2$	$x < 1 \vee x > 3$
INTERSEZIONI CON ASSE X	$(-2;0)$	$(1;0)$ $(3;0)$
INTERSEZIONI CON ASSE Y	$(0,2)$	$(0;2)$
FUNZIONE CRESCENTE	$f'(x) > 0$ $x < -1$	$x < -1 \vee x > 2$





DOMINIO $x \geq -1$ | $x < 1 \vee 1 < x \leq 4$

CODOMINIO $y \leq 3$ | $y \leq 1$

SEGNO $y > 0$ $-1 < x < 2$ | $2 < x < 3$

INTERSEZIONI CON ASSE X $(-1;0)$ $(2;0)$ | $(2;0)$ $(4;0)$

INTERSEZIONI CON ASSE Y $(0;3)$ | $(0;-1)$

FUNZIONE CRESCENTE $y' > 0$ $-1 < x < 0$ | $1 < x < 3$