

ESERCIZI svolti: SOMMA di radicali quadratici

$$S1) \quad 6\sqrt{2} - 7\sqrt{2} + 10\sqrt{2}$$

$$S2) \quad 14\sqrt{x} - 6\sqrt{y} + 16\sqrt{x} - 5\sqrt{y} - \sqrt{x}$$

$$S3) \quad 3\sqrt{50} - 2\sqrt{18} + 4\sqrt{8}$$

$$S4) \quad 4\sqrt{25x} - \sqrt{9x} + 8\sqrt{4x} + \sqrt{49x}$$

$$S5) \quad 7\sqrt{9x} - 2\sqrt{25x} + \sqrt{36x}$$

-----soluzioni-----

$$S1) \quad 6\sqrt{2} - 7\sqrt{2} + 10\sqrt{2} = (6 - 7 + 10)\sqrt{2} = 9\sqrt{2}$$

S2)

$$14\sqrt{x} - 6\sqrt{y} + 16\sqrt{x} - 5\sqrt{y} - \sqrt{x} = (14 + 16 - 1)\sqrt{x} + (-6 - 5)\sqrt{y} = 29\sqrt{x} - 11\sqrt{y}$$

S3)

$$\begin{aligned} 3\sqrt{50} - 2\sqrt{18} + 4\sqrt{8} &= 3\sqrt{2 \cdot 5^2} - 2\sqrt{2 \cdot 3^2} + 4\sqrt{2^3} = \\ 3 \cdot 5\sqrt{2} - 2 \cdot 3\sqrt{2} + 4 \cdot 2\sqrt{2} &= 15\sqrt{2} - 6\sqrt{2} + 8\sqrt{2} = \\ &= 17\sqrt{2} \end{aligned}$$

S4)

$$\begin{aligned} 4\sqrt{25x} - \sqrt{9x} + 8\sqrt{4x} + \sqrt{49x} &= 4\sqrt{5^2x} - \sqrt{3^2x} + 8\sqrt{2^2x} + \sqrt{7^2x} = \\ 4 \cdot 5\sqrt{x} - 3\sqrt{x} + 8 \cdot 2\sqrt{x} + 7\sqrt{x} &= 20\sqrt{x} - 3\sqrt{x} + 16\sqrt{x} + 7\sqrt{x} = \\ (20 - 3 + 16 + 7)\sqrt{x} &= 40\sqrt{x} \end{aligned}$$

S5)

$$\begin{aligned} 7\sqrt{9x} - 2\sqrt{25x} + \sqrt{36x} &= 7\sqrt{3^2x} - 2\sqrt{5^2x} + \sqrt{6^2x} = \\ 7 \cdot 3\sqrt{x} - 2 \cdot 5\sqrt{x} + 6\sqrt{x} &= 21\sqrt{x} - 10\sqrt{x} + 6\sqrt{x} = \\ (21 - 10 + 6)\sqrt{x} &= 17\sqrt{x} \end{aligned}$$

ESERCIZI svolti: espressioni con radicali quadratici

$$E1) \quad 3\sqrt{2}(\sqrt{3} - 4\sqrt{2}) + \sqrt{2} \cdot (3\sqrt{2} - 5\sqrt{3})$$

$$E2) \quad 4\sqrt{x} \cdot (3\sqrt{y} + 2\sqrt{x}) - 5\sqrt{y} \cdot (2\sqrt{x} - 4\sqrt{y}) - \sqrt{x} \cdot (-\sqrt{y})$$

$$E3) \quad 3\sqrt{5} \cdot (2\sqrt{5} - \sqrt{10}) + 4\sqrt{5} \cdot (\sqrt{5} + 4\sqrt{10}) =$$

$$E4) \quad 2\sqrt{6} \cdot (4\sqrt{3} - 2\sqrt{2} + \sqrt{6}) + 5\sqrt{3} \cdot (\sqrt{6} - \sqrt{3}) =$$

-----soluzioni-----

E1)

$$3\sqrt{2}(\sqrt{3} - 4\sqrt{2}) + \sqrt{2} \cdot (3\sqrt{2} - 5\sqrt{3}) =$$

$$3\sqrt{6} - 12\sqrt{4} + 3\sqrt{4} - 5\sqrt{6} =$$

$$-2\sqrt{6} - 9\sqrt{4} = -2\sqrt{6} - 9 \cdot 2 = -2\sqrt{6} - 18$$

E2)

$$4\sqrt{x} \cdot (3\sqrt{y} + 2\sqrt{x}) - 5\sqrt{y} \cdot (2\sqrt{x} - 4\sqrt{y}) - \sqrt{x} \cdot (-\sqrt{y}) =$$

$$= 12\sqrt{xy} + 8\sqrt{x^2} - 10\sqrt{xy} + 20\sqrt{y^2} + \sqrt{xy} = 3\sqrt{xy} + 8x + 20y$$

E3)

$$3\sqrt{5} \cdot (2\sqrt{5} - \sqrt{10}) + 4\sqrt{5} \cdot (\sqrt{5} + 4\sqrt{10}) =$$

$$= 6\sqrt{25} - 3\sqrt{50} + 4\sqrt{25} + 16\sqrt{50} = 10\sqrt{25} + 13\sqrt{50} =$$

$$\text{scompongo e porto fuori} = 10\sqrt{5^2} + 13\sqrt{5^2 \cdot 2} =$$

$$10 \cdot 5 + 13 \cdot 5\sqrt{2} = 50 + 65\sqrt{2}$$

E4)

$$2\sqrt{6} \cdot (4\sqrt{3} - 2\sqrt{2} + \sqrt{6}) + 5\sqrt{3} \cdot (\sqrt{6} - \sqrt{3}) =$$

$$12\sqrt{18} - 4\sqrt{12} + \sqrt{36} + 5\sqrt{18} - 5\sqrt{9} = 17\sqrt{18} - 4\sqrt{12} + \sqrt{36} - 5\sqrt{9} =$$

$$\text{scompongo e porto fuori} = 17\sqrt{3^2 \cdot 2} - 4\sqrt{2^2 \cdot 3} + \sqrt{6^2} - 5\sqrt{3^2} =$$

$$= 17 \cdot 3\sqrt{2} - 4 \cdot 2\sqrt{3} + 6 - 5 \cdot 3 = 51\sqrt{2} - 8\sqrt{3} + 6 - 15 = 51\sqrt{2} - 8\sqrt{3} - 9$$

ESERCIZI svolti: prodotti notevoli con radicali

da ricordare: il quadrato di una radice quadratica = radicando !

$$(\sqrt{a})^2 = \sqrt{a} \cdot \sqrt{a} = a$$

esempi: $(\sqrt{2})^2 = 2$; $(\sqrt{3})^2 = 3$; $(\sqrt{4})^2 = 4$; $(\sqrt{5})^2 = 5$

$(3\sqrt{2})^2 = \text{elevo_il_coefficiente_e_la_radice} = (3)^2 \cdot (\sqrt{2})^2 = 9 \cdot 2 = 18$

$(5\sqrt{3})^2 = 25 \cdot 3 = 75$; $(4\sqrt{5})^2 = 16 \cdot 5 = 80$; $(2\sqrt{7})^2 = 4 \cdot 7 = 28$

REGOLE PRODOTTI NOTEVOLI

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$$

$$(\sqrt{a} \pm \sqrt{b})^2 = a + b \pm 2\sqrt{ab}$$

esercizi

P1) $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$; $(\sqrt{7} + 3)(\sqrt{7} - 3)$; $(6 - \sqrt{11})(6 + \sqrt{11})$

P2) $(2\sqrt{5} + \sqrt{3})(2\sqrt{5} - \sqrt{3})$; $(3\sqrt{7} + 5)(3\sqrt{7} - 5)$; $(5\sqrt{3} - 2\sqrt{7})(5\sqrt{3} + 2\sqrt{7})$

P3) $(\sqrt{3} + \sqrt{2})^2$; $(\sqrt{3} - 2\sqrt{5})^2$; $(5\sqrt{3} - \sqrt{6})^2$

-----soluzioni-----

$$(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$$

P1) $(\sqrt{7} + 3)(\sqrt{7} - 3) = (\sqrt{7})^2 - (3)^2 = 7 - 9 = -2$

$$(6 - \sqrt{11})(6 + \sqrt{11}) = (6)^2 - (\sqrt{11})^2 = 36 - 11 = 25$$

$$(2\sqrt{5} + \sqrt{3})(2\sqrt{5} - \sqrt{3}) = (2\sqrt{5})^2 - (\sqrt{3})^2 = 20 - 3 = 17$$

P2) $(3\sqrt{7} + 5)(3\sqrt{7} - 5) = (3\sqrt{7})^2 - (5)^2 = 63 - 25 = 38$

$$(5\sqrt{3} - 2\sqrt{7})(5\sqrt{3} + 2\sqrt{7}) = (5\sqrt{3})^2 - (2\sqrt{7})^2 = 75 - 28 = 47$$

$$(\sqrt{3} + \sqrt{2})^2 = 3 + 2 + 2\sqrt{6} = 5 + 2\sqrt{6}$$

$$(\sqrt{3} - 2\sqrt{5})^2 = 3 + 20 - 4\sqrt{15} = 23 - 4\sqrt{15}$$

P3) $(5\sqrt{3} - \sqrt{6})^2 = 75 + 6 - 10\sqrt{18} = 81 - 10\sqrt{18} = 81 - 10\sqrt{3^2 \cdot 2} =$
 $= 81 - 30\sqrt{2}$

ESERCIZI svolti: razionalizza il denominatore

$$R1) \quad \frac{5x}{\sqrt{2}}; \frac{4a}{3\sqrt{7a}}; \frac{3ab}{2\sqrt{3b}}$$

$$R2) \quad \frac{10x}{\sqrt{7}-\sqrt{2}}; \frac{3a}{\sqrt{11}+5}; \frac{21}{3-\sqrt{2}}$$

$$R3) \quad \frac{6}{2\sqrt{3}+\sqrt{5}}; \frac{7x}{5\sqrt{2}-3\sqrt{2}}; \frac{13a}{2\sqrt{5}+3\sqrt{7}}$$

-----soluzioni-----

R1)

$$\frac{5x}{\sqrt{2}} = \frac{5x}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5x\sqrt{2}}{2}$$

$$\frac{4a}{3\sqrt{7a}} = \frac{4a}{3\sqrt{7a}} \cdot \frac{\sqrt{7a}}{\sqrt{7a}} = \frac{4a\sqrt{7a}}{3 \cdot 7a} = \frac{4a\sqrt{7a}}{21a} = \frac{4\sqrt{7a}}{21}$$

$$\frac{3ab}{2\sqrt{3b}} = \frac{3ab}{2\sqrt{3b}} \cdot \frac{\sqrt{3b}}{\sqrt{3b}} = \frac{3ab\sqrt{3b}}{2 \cdot 3b} = \frac{3ab\sqrt{3b}}{6b} = \frac{a\sqrt{3b}}{2}$$

R2)

$$\frac{10x}{\sqrt{7}-\sqrt{2}} = \frac{10x}{(\sqrt{7}-\sqrt{2})(\sqrt{7}+\sqrt{2})} \cdot (\sqrt{7}+\sqrt{2}) = \frac{10x(\sqrt{7}+\sqrt{2})}{7-2} = \frac{10x(\sqrt{7}+\sqrt{2})}{5} = 2x(\sqrt{7}+\sqrt{2})$$

$$\frac{8a}{\sqrt{11}+5} \cdot \frac{(\sqrt{11}-5)}{(\sqrt{11}-5)} = \frac{8a(\sqrt{11}-5)}{11-25} = \frac{8a(\sqrt{11}-5)}{-14} = -\frac{4a(\sqrt{11}-5)}{7} = \frac{4a(5-\sqrt{11})}{7}$$

$$\frac{21}{(3-\sqrt{2})(3+\sqrt{2})} \cdot (3+\sqrt{2}) = \frac{21(3+\sqrt{2})}{9-2} = \frac{21(3+\sqrt{2})}{7} = 3(3+\sqrt{2})$$

R3)

$$\frac{14}{2\sqrt{3}+\sqrt{5}} \cdot \frac{(2\sqrt{3}-\sqrt{5})}{(2\sqrt{3}-\sqrt{5})} = \frac{14(2\sqrt{3}-\sqrt{5})}{(2\sqrt{3})^2 - (\sqrt{5})^2} = \frac{14(2\sqrt{3}-\sqrt{5})}{12-5} = \frac{14(2\sqrt{3}-\sqrt{5})}{7} = 2(2\sqrt{3}-\sqrt{5})$$

$$\frac{4x}{5\sqrt{2}-3\sqrt{2}} \cdot \frac{(5\sqrt{2}+3\sqrt{2})}{(5\sqrt{2}+3\sqrt{2})} = \frac{4x(5\sqrt{2}+3\sqrt{2})}{(5\sqrt{2})^2 - (3\sqrt{2})^2} = \frac{4x(5\sqrt{2}+3\sqrt{2})}{50-18} = \frac{4x(3\sqrt{2}+5\sqrt{2})}{32} = \frac{x(3\sqrt{2}+5\sqrt{2})}{8}$$

$$\frac{13a}{2\sqrt{5}+\sqrt{7}} \cdot \frac{(2\sqrt{5}-\sqrt{7})}{(2\sqrt{5}-\sqrt{7})} = \frac{13a(2\sqrt{5}-\sqrt{7})}{(2\sqrt{5})^2 - (\sqrt{7})^2} = \frac{13a(2\sqrt{5}-\sqrt{7})}{20-7} = \frac{13a(2\sqrt{5}-\sqrt{7})}{13} = a(2\sqrt{5}-\sqrt{7})$$